Multiphase Flow and Heat Transfer

ME546

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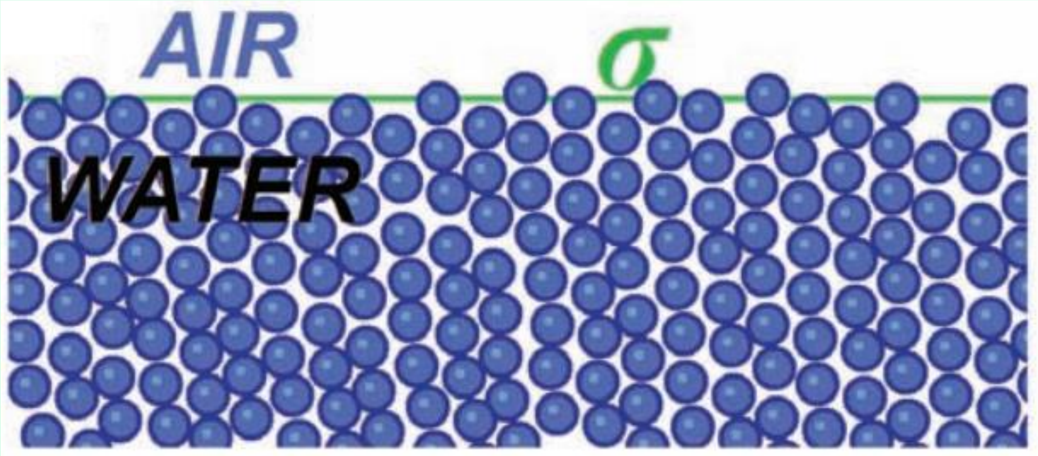
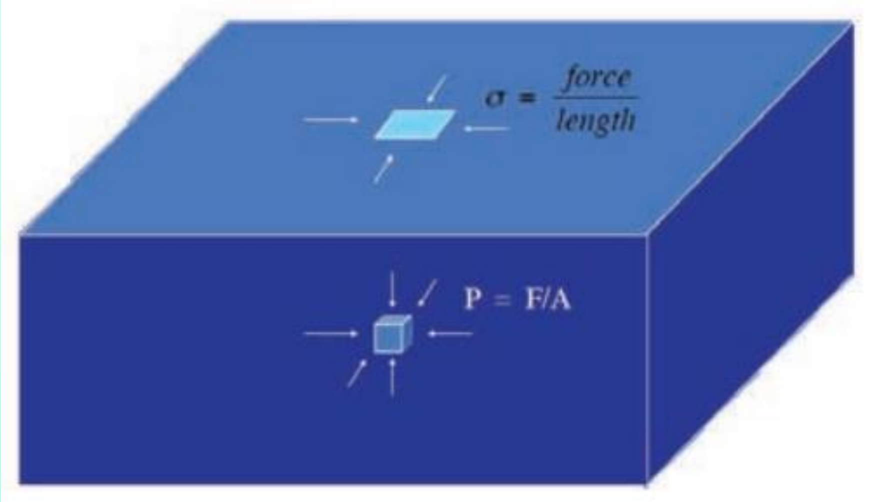
Surface Tension

The free surface between air and water at a molecular scale

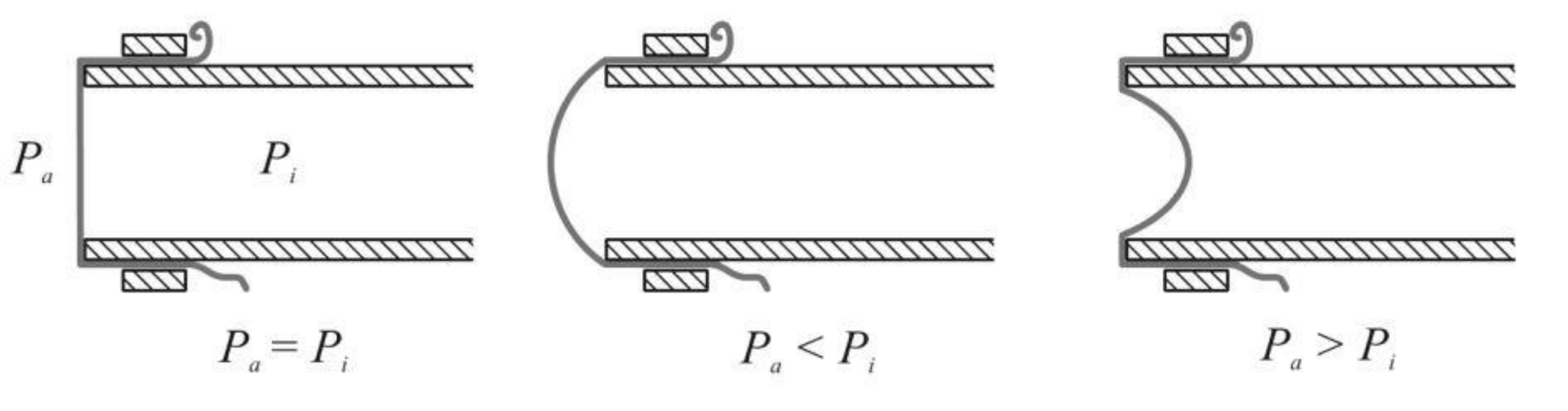


Molecules sitting at a free liquid surface against vacuum or gas have weaker binding than molecules in the bulk.

Surface tension is analogous to a negative surface pressure



Surface Tension

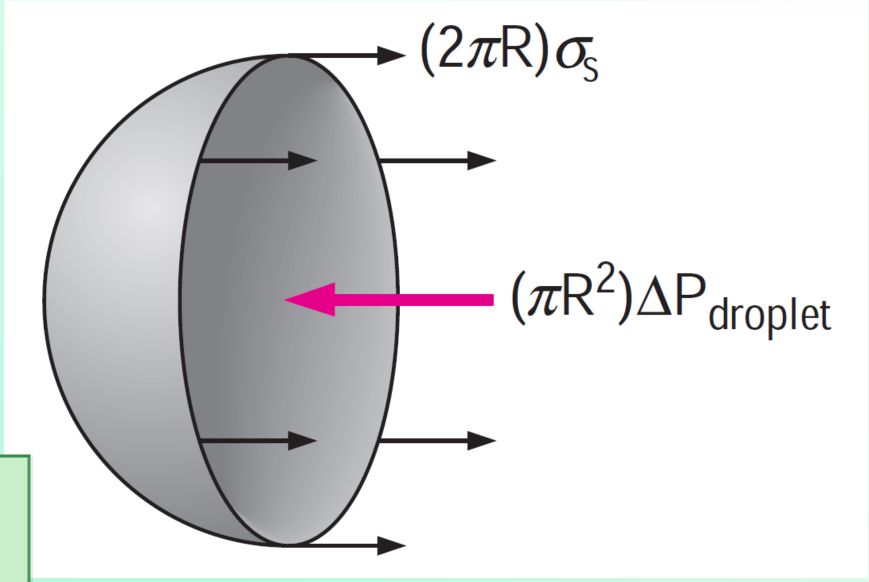


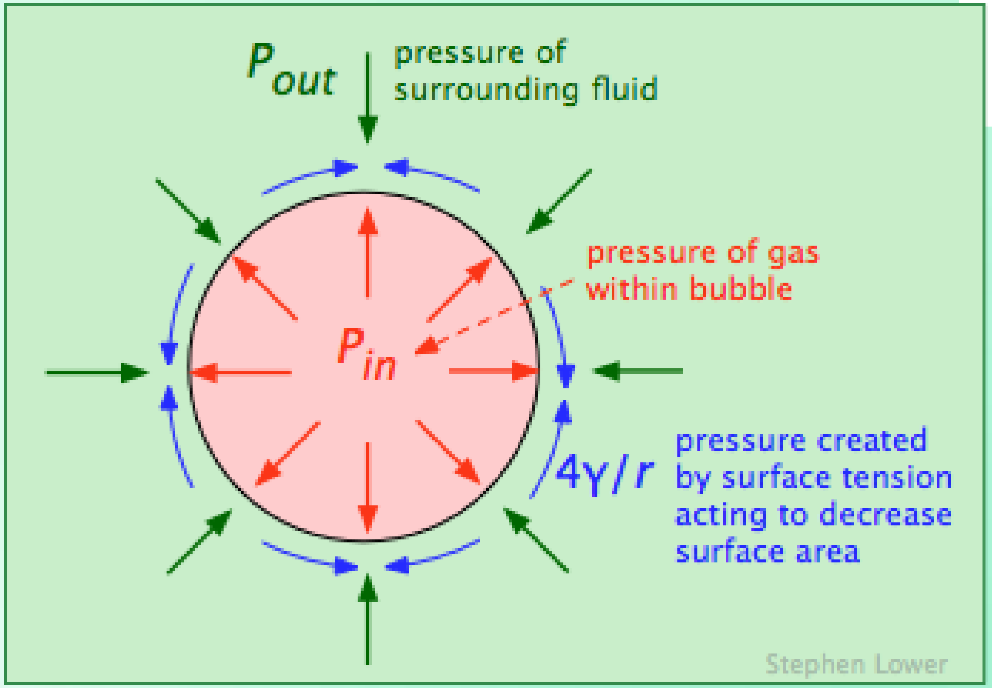
Rubber membrane at the end of a cylindrical tube

An inner pressure P_i can be applied, which is different than the outside pressure P_a.

Young-Laplace Equation

Surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet.





Young-Laplace Equation

Consider a differential increase in the radius of the droplet due to the addition of a differential amount of mass.

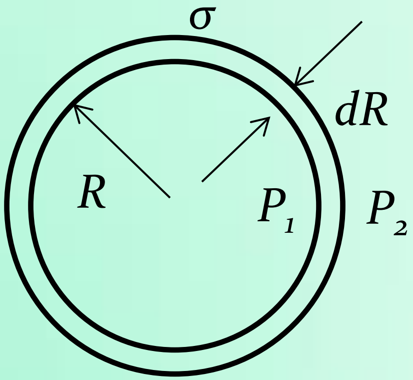
Surface tension is the increase in the surface energy per unit area.

The increase in the surface energy of the droplet during the differential expansion process:

$$\delta W_{\text{surface}} = \sigma\, dA$$

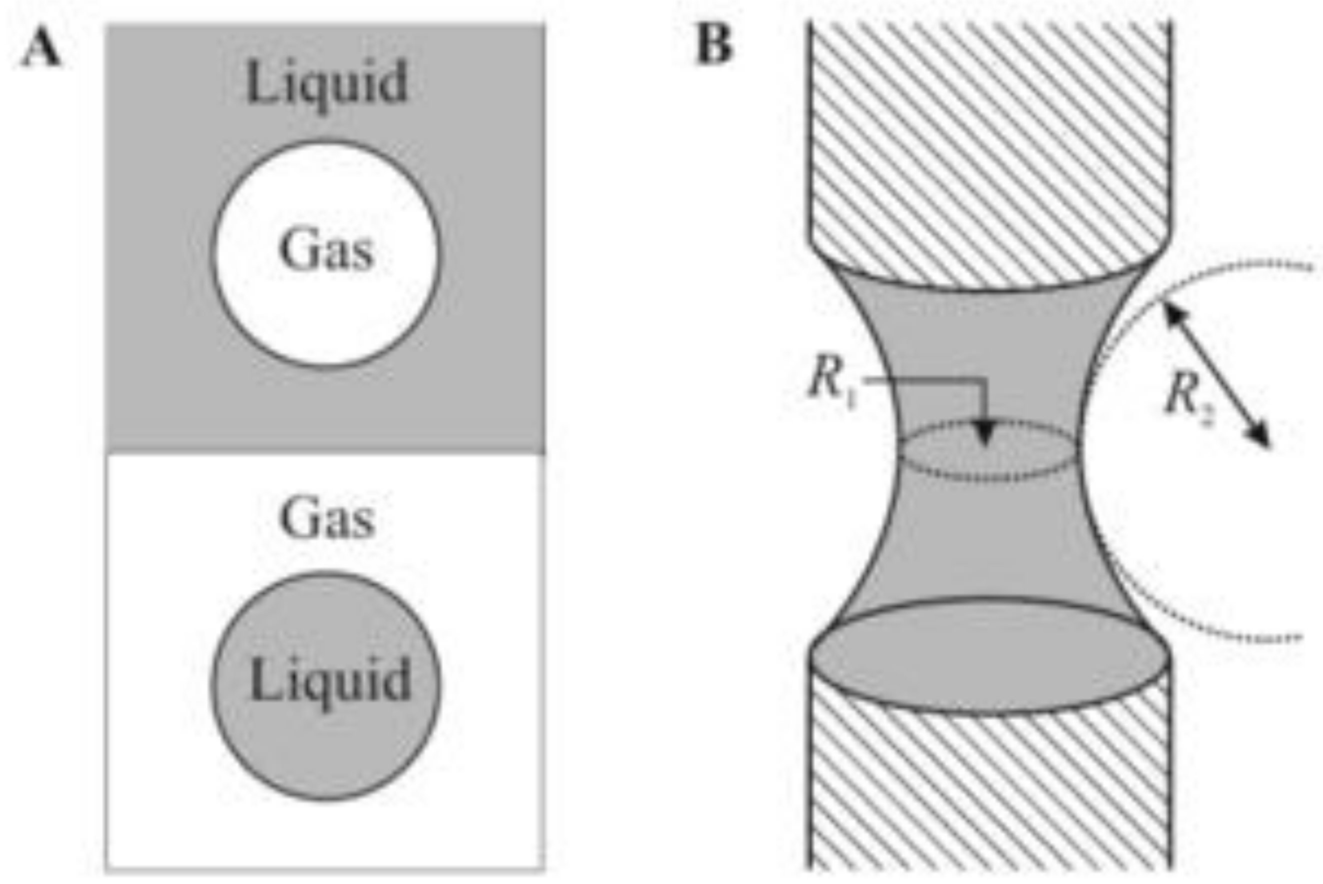
$$\delta W_{\text{expansion}} = FdR = \Delta PAdR = \Delta PdV$$

$$\Delta P = P_1 - P_2 = \frac{2\sigma}{R}$$



P_1 - P_2 is positive and so $P_1 > P_2$

Young-Laplace Equation



A liquid meniscus with radii of curvature of opposite sign between two solid cylinders

Young-Laplace Equation

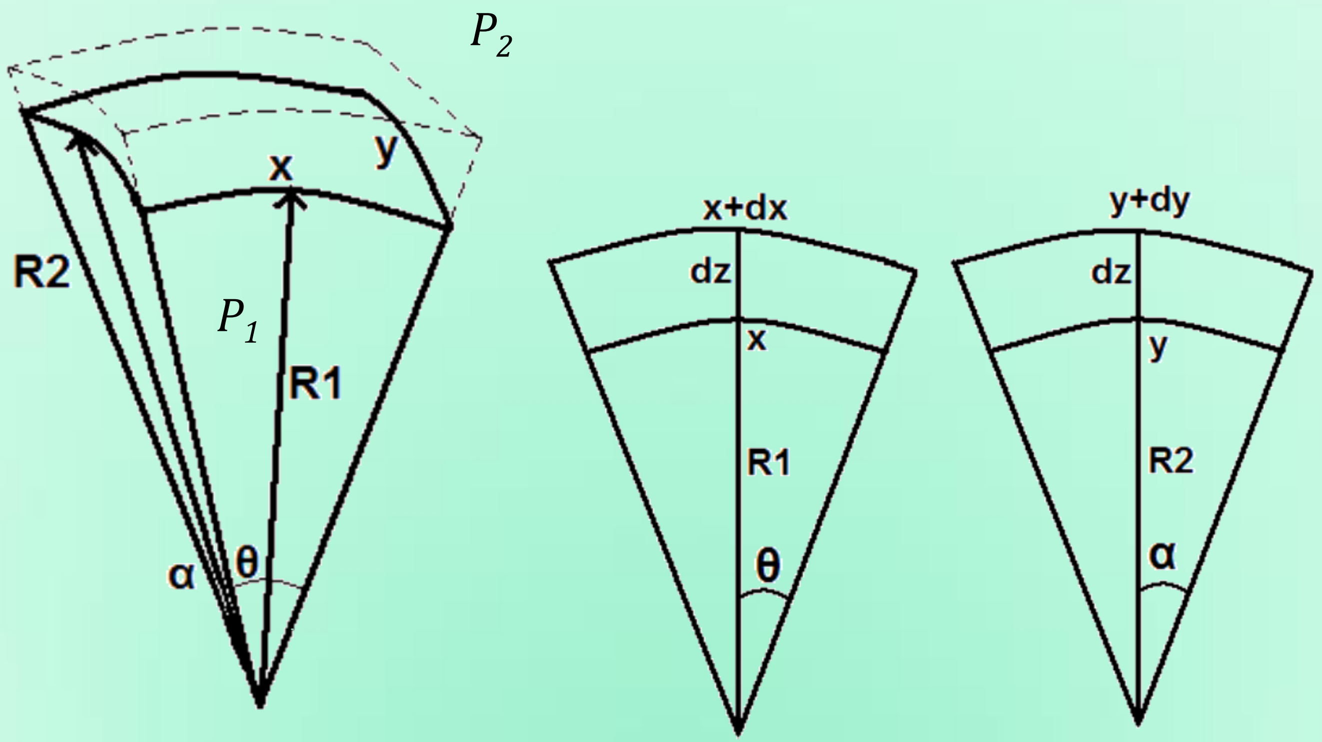
$$dA = (x + dx)(y + dy) - xy = xdy + ydx$$

$$\delta W_{\text{surface}} = \sigma\, dA = \sigma(xdy + ydx)$$

$$\delta W_{\text{expansion}} = \Delta P A dz = \Delta P xydz$$

$\Delta P = P_1 - P_2$

P_1 is on concave side



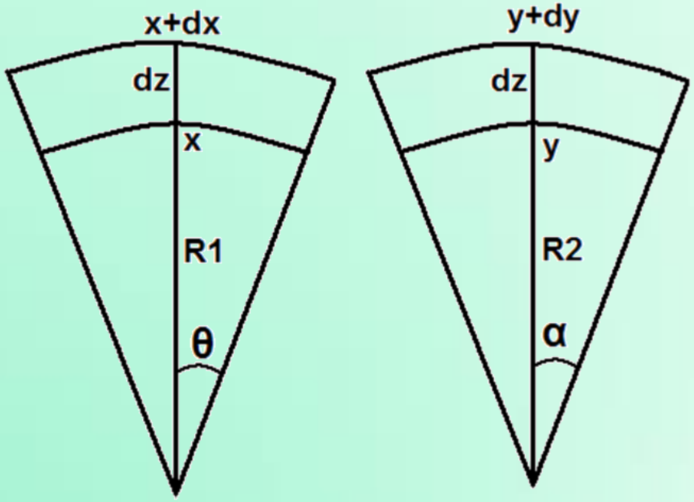
Young-Laplace Equation

$$\Delta P\, xy\, dz = \sigma\left(xdy + ydx\right)$$

$$\frac{x+dx}{x} = \frac{R_1 + dz}{R_1}$$

$$\frac{y+dy}{y} = \frac{R_2 + dz}{R_2}$$

$$\Delta P = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



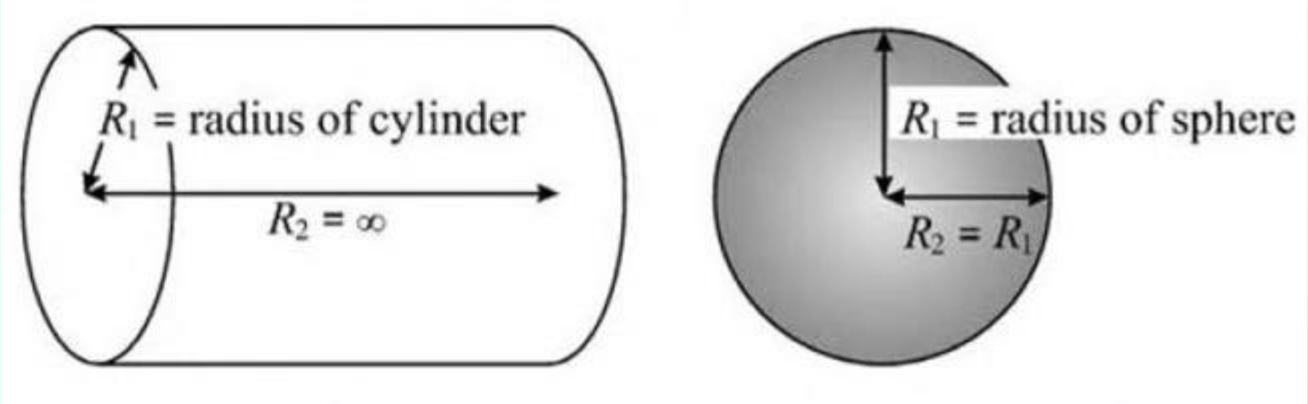
Capillary pressure difference,

$$\Delta P = P_1 - P_2$$

Pressure on the concave side is higher

Young-Laplace Equation

The planes defining the radii of curvature must be perpendicular to each other and contain the surface normal.



For a cylinder of radius R a convenient choice is

$R_1 = R$ and $R_2 = \infty$ so that the curvature is $1/R$.

For a sphere with radius R we have

$R_1 = R_2$ and the curvature is $2/R$.

Young-Laplace Equation

Compare a spherical bubble with a diameter of 1 mm and a bubble of 10 nm diameter in pure water. σ = 0.072 N/m

$$\Delta P_{\mathrm{1mm}} = 288\,\mathrm{Pa}$$

$$\Delta P_{\mathrm{1nm}} = 28.8\,\mathrm{MPa}$$

$$P_{\mathrm{inside,\ 1nm}} = 28.8\,\mathrm{MPa} + 0.1\,\mathrm{MPa} = 28.9\,\mathrm{MPa}$$

Young-Laplace Equation

If we know the shape of a liquid surface we know its curvature and we can calculate the pressure difference.

In the absence of external fields (e.g., gravity), the pressure is same everywhere in the liquid; otherwise there would be a flow of liquid of regions of low pressure.

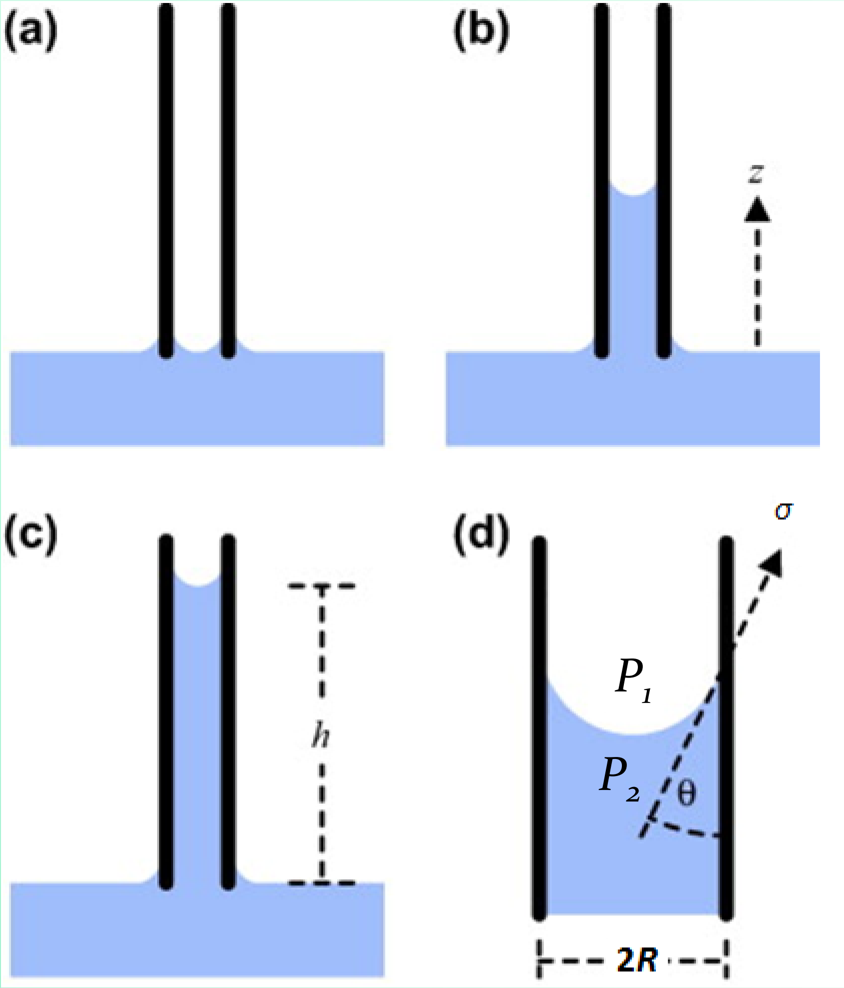
Thus, ΔP is constant and Young-Laplace equation tells us in this case the surface of the liquid has the same curvature everywhere.

It is possible to calculate the equilibrium shape of a liquid surface.

If we know the pressure difference and some boundary conditions (such as volume of the liquid and its contact line) we can calculate the geometry of the liquid surface.

Capillary Rise or Depression

Why makes the liquid column to rise/fall?



Capillary Rise or Depression

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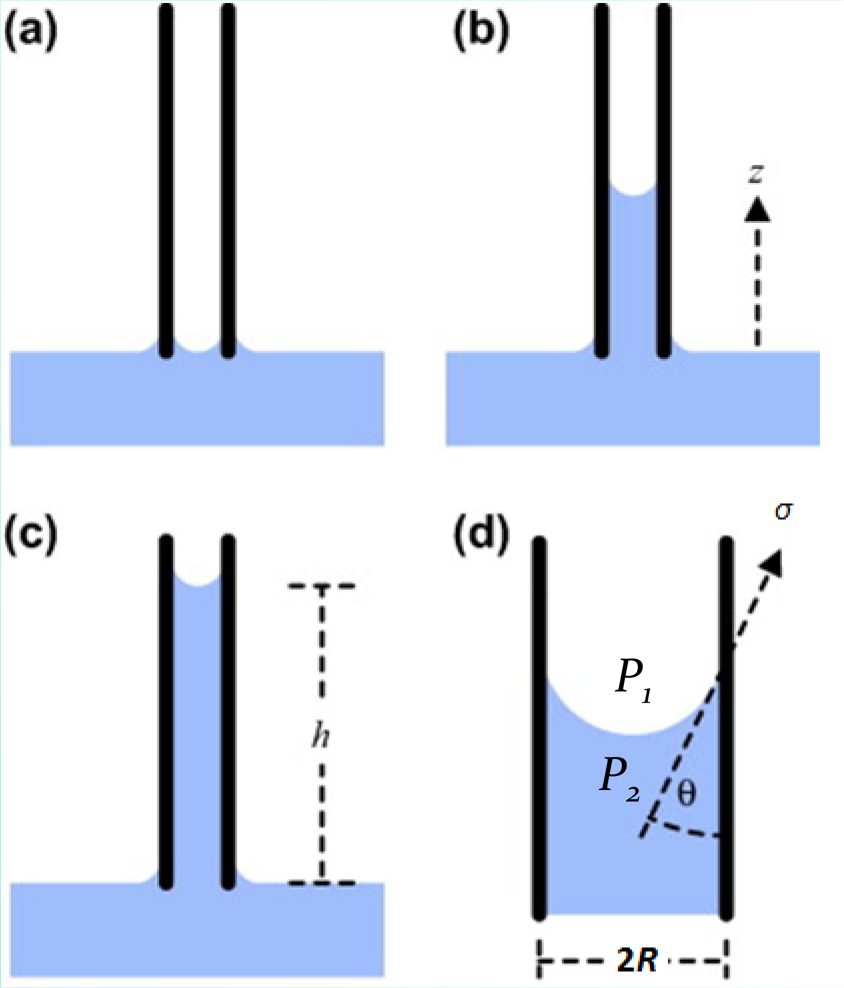
To satisfy Young-Laplace equation.



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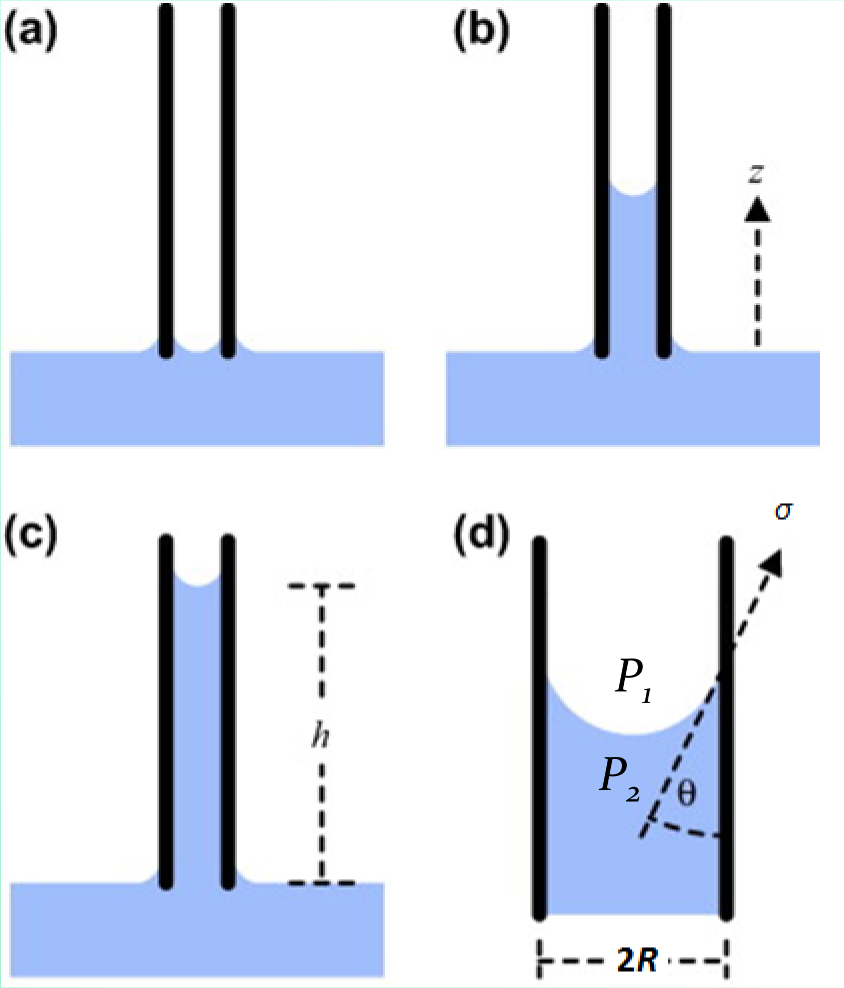


Let us assume that the meniscus is of spherical of radius, *a*.

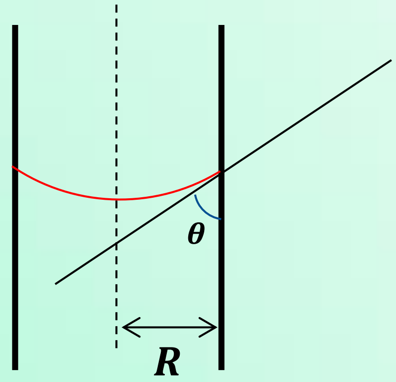
$$P_1 - P_2 = \frac{2\sigma}{a}$$

From hydrostatics:

$$P_2 = P_1 + \rho_v gh - \rho_l gh$$

$$P_1 - P_2 = (\rho_l - \rho_v) gh$$

Capillary Rise or Depression



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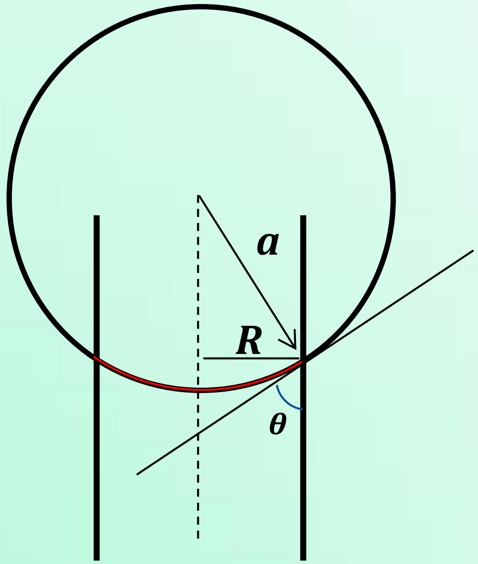
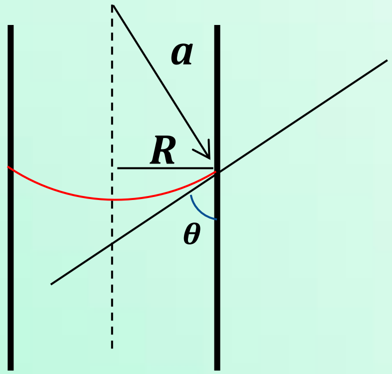


Capillary Rise or Depression

$$\cos\theta = \frac{R}{a}$$

Radius of the meniscus:



$$a = \frac{R}{\cos\theta}$$

$$P_1 - P_2 = \frac{2\sigma}{a} = \frac{2\sigma\cos\theta}{R} = \left(\rho_l - \rho_v\right)gh$$

Height of the capillary rise:

$$h = \frac{2\sigma\cos\theta}{\left(\rho_l - \rho_v\right)gR}$$

Capillary Rise or Depression

$$h = \frac{2\sigma\cos\theta}{(\rho_l - \rho_v)gR}$$

- It suggests that every point on the meniscus is at the same height *h* from the surface of the liquid reservoir, or in other words, the meniscus is flat!
- A more accurate derivation should consider the deviation of meniscus spherical shape in view of the elevation of each point above the flat surface of the liquid. This involves the solution of the general Young-Laplace equation using the expressions for R_1 and R_2.

Characteristic length scale:

$$L_c = \sqrt{\frac{\sigma}{(\rho_l - \rho_v)g}}$$

Capillary Rise or Depression

Compare a capillary rise with the tube diameters of 5 mm, 1 mm (laboratory test tube) and 200 nm (capillary diameter of a Redwood tree) in pure water at 20°C. The contact angle of the interface with the tube wall is 20°.

σ = 0.0728 N/m

$$h = \frac{2\sigma\cos\theta}{(\rho_l - \rho_v)gR}$$

d = 5 mm, h = 5.6 mm, P_{water} = 101.3 kPa

d = 1 mm, h = 0.03 m, P_{water} = 101 kPa

d = 200 nm, h = 140 m, P_{water} = -1.3 MPa

Negative Pressure?